DETERMINATION OF ELECTRICAL CONDUCTIVITY PROFILES FROM MULTI-FREQUENCY IMPEDANCE MEASUREMENTS

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INTRODUCTION

Most existing eddy current methods implicitly assume uniform electrical conductivity throughout the sample. However, eddy current measurements recorded over a range of frequencies extract conductivity information over a range of depths, and, thus, are sensitive to spatial variations in conductivity. Determination of spatial profiles of conductivity offers the potential of a new technique in metals processing, where variations in conductivity may arise in composite materials or from non-uniform temperature distributions. We describe here a conductivity profiling method based on an iterative nonlinear least-squares algorithm that operates on multi-frequency impedance data.

The idea of multi-frequency eddy current profiling is based on the skin effect principle, which states that an AC magnetic field penetrates a conductor approximately one skin depth, $\delta = 1/\sqrt{\pi\mu\sigma f}$, where μ is permeability, σ is conductivity and f is frequency. Since δ varies inversely with the the square root of frequency, eddy current measurements performed over a range of frequencies should permit one to probe the sample over a range of depths.

In the work reported in this paper, cylindrical metallic samples were used in which the conductivity was assumed to vary only radially. In tests of the conductivity profiling algorithm, a simple two-layer axisymmetric conductivity model was assumed. This model can be described as a solid rod, or "pin", of uniform conductivity surrounded by a cylindrical layer, or "sleeve", of uniform but differing conductivity. We refer to this as the pin-in-sleeve model. In addition to the conductivity values, the radius of the boundary between the pin and sleeve was also regarded as an unknown parameter. We thus assume a three parameter model: the conductivities of the pin and sleeve, and the radius of the pin (or equivalently the inside sleeve radius).

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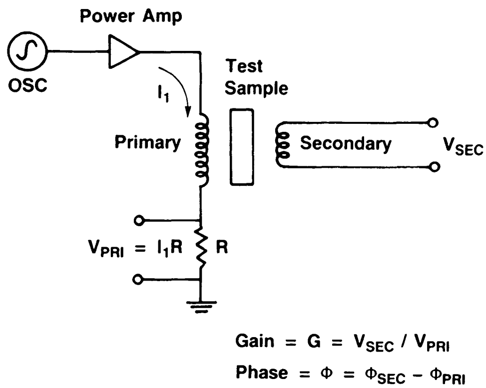

Fig. 1. Schematic diagram of the dual-coil measurement system.

EXPERIMENT

The measurement apparatus was composed of the dual coil arrangement shown in Fig. 1. The primary coil encloses a secondary coil, and both surround the cylindrical sample. The recorded impedance is the ratio of the voltage induced in the secondary coil to the current driving the primary coil. A Hewlett-Packard impedance analyzer was used to record impedance measurements at 23 frequencies, ranging from 27 Hz to 5000 Hz in logarithmic increments. The uniform exciting magnetic field was generated by a primary coil of diameter 1.7 in., length 14 in. and wound with no. 24 copper wire. The secondary coil, of diameter 1.44 in. with 50 turns of closely wound no. 30 enameled copper wire, recorded the response of the sample. The impedance analyzer was configured to measure the magnitude and phase of the secondary voltage relative to the current in the primary coil.

The dual coil method provides several advantages over a single coil impedance system. (1) The primary coil can be made long compared to the secondary coil and the sample, thus producing a more uniform field and reducing fringing effects. (2) A power amplifier can be inserted into the system to boost the primary current at low frequencies. (3) The transfer impedance measurement is primarily dependent on the geometry of the coils and insensitive to the resistance of the coils.

THEORY

The reconstruction algorithm was based on minimizing the mean-square error,

$$E(\vec{p}) = \sum_{m=1}^{M} \left| Z(\omega_m) - \hat{Z}(\omega_m, \vec{p}) \right|^2 \quad , \tag{1}$$

with respect to $\vec{p}$, where $\vec{p}$ is a parameter vector defining the conductivity profile, $Z(\omega)$ is the measured impedance at frequency ω, $\hat{Z}(\omega,\vec{p})$ is the computed impedance at frequency ω based on the profile specified by $\vec{p}$, and M is the number of frequency measurements. $\hat{Z}(\omega,\vec{p})$ is the solution to the "forward problem", that is, the predicted impedance given the conductivity profile defined by $\vec{p}$. In the two-layer problem considered here, $\vec{p}$ consists of three components: the conductivities of the pin and sleeve and the pin radius. A more detailed description of the least-squares algorithm and an analytical solution to the forward problem for an axisymmetric layered sample can be found in ref. 1.

RECONSTRUCTIONS FROM EXPERIMENTAL MEASUREMENTS

Three experimental cases were considered: the pin alone, the sleeve alone, and the pin-sleeve combination. The reconstruction algorithm was written assuming a two-layer (i.e., pin-sleeve) model with two unknown conductivities and an unknown pin radius. As a first example, impedance data were recorded with the pin alone (without the sleeve) inserted into the system. In this problem, the algorithm had no knowledge of the fact that the "sleeve" in this case was air (of zero conductivity). The resulting reconstruction is shown in Fig. 2 as the dashed line, where the solid line indicates the true conductivity profile of the pin. Note that the algorithm correctly predicted the "sleeve" (the region outside the pin) to have zero conductivity. In a second problem, data were recorded when the sleeve alone was placed in the system without the pin, and the reconstruction of the sleeve is shown as the dashed line in Fig. 3, where again the true profile is indicated by the solid line. Here the algorithm correctly predicted the "pin" to have zero conductivity (that of air). In the separate reconstructions of the pin and sleeve, the location of the interface (the pin radius) was accurately determined and

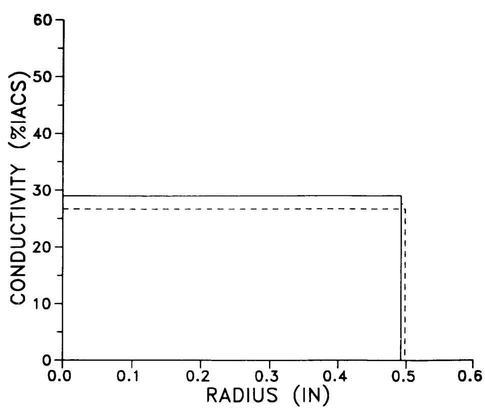

Fig. 2. Reconstruction (dashed line) of the conductivity profile of a pin (solid line) with no sleeve.

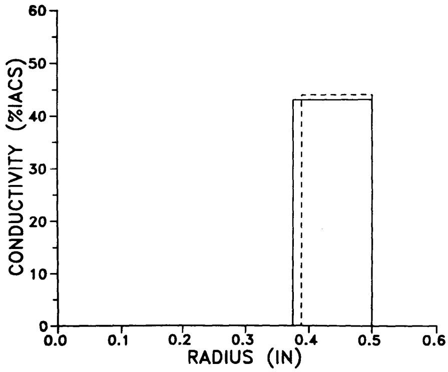

Fig. 3. Reconstruction (dashed line) of the conductivity profile of a sleeve (solid line) with the pin removed.

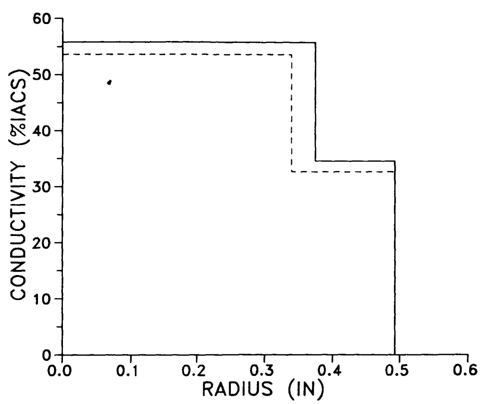

Fig. 4. Reconstruction (dashed line) of the conductivity profile of a pin in sleeve (solid line).

the conductivity values were within several percent IACS of their true values. In a third example, measurements were recorded with the pin inserted into the sleeve, and the actual and reconstructed profiles are indicated in Fig. 4 by the solid and dashed lines, respectively. In the reconstruction of the pin-sleeve combination, the conductivity values and the pin radius were in reasonably close agreement with the actual values, but were determined somewhat less accurately than the separate pin and sleeve reconstructions. We remark that the "true" conductivities indicated by the solid lines in these figures were measured by a commercial conductivity meter which has an inherent uncertainty of one to two percent IACS.

CONCLUSION

The discrepancies between the reconstructions and the true profiles are probably a result of a variety of simplifying approximations employed in deriving the forward solution. For example, the idealized forward calculation of the impedance assumes a perfectly uniform magnetic field produced by the primary coil in the absence of the sample, where in fact small field inhomogeneities may exist. Also, fringing effects are assumed negligible in the forward solution, which in practice may not be entirely absent. Other effects may also exist that result in an imperfect model of the measurement process. Further work is needed to establish the causes of the deviations of real measurements from those predicted on the basis of the idealized forward calculation.

Despite these problems, the above reconstructions derived from experimental data demonstrate the potential of multi-frequency eddy current systems for recovering simple spatially varying conductivity profiles.

REFERENCE

1. S. J. Norton, A. H. Kahn, and M. L. Mester, Research in Nondestructive Evaluation (in press).